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INTUITIONISTIC FUZZY SOMEWHERE DENSE SETS

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ABSTRACT

In this paper, a new class of Intuitionistic fuzzy somewhere dense set has been defined and studied. Some charactererization has been obtained. Further Intuitionistic fuzzy somewhere continuous function has been introduced and some of its properties are studied.

KEYWORDS: Intuitionistic Fuzzy Somewhere Dense Set, Intuitionistic Fuzzy Somewhere Continuous Function, Intuitionistic Fuzzy Simply * Continuous Function, Intuitionistic Fuzzy cs Dense Set, Intuitionistic Fuzzy Hyper Connected Space, Intuitionistic Fuzzy P- Space & Intuitionistic Fuzzy Submaximal Space

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Original Article

1. INTRODUCTION AND PRELIMINARIES

In Zadeh [5] introduced the fundamental concept of a fuzzy set. Chang extended the concept of point set topology to fuzzy sets. Atanassov [2] introduced intuitionistic fuzzy set. After the introduction of intuitionistic fuzzy topology by Cocker [3] in 1997. T. M. Al – Shami [1] introduced a new class of sets, namely Somewhere dense sets in Topological Spaces.

2. INTUITIONISTIC FUZZY SOMEWHERE DENSE SETS

Definition: 2.1

Let (Y,T) be an intuitionistic fuzzy topological space (briefly. IFTS). An intuitionistic fuzzy set σ defined on X is called an intuitionistic fuzzy (briefly, IF) somewhere dense set if intcl(σ)≠0 in (Y,T). That is, σ is an IF some where dense set in (Y,T) if there exists a non - zero IFopen set γ in (Y,T) such that γ ≤cl(σ).

Definition: 2.1

If σ is an IFsomewhere dense set in an IFTS (Y,T), then 1- σ is called an IF complement of intuitionistic fuzzy somewhere dense set in (Y,T). It is to be denoted as IF cs dense set in (Y,T).

Example: 2.1

Let X={a,b,c} and A and B are IF subsets in X, where

A $=< x,(0.3,0.1,0.4),(0.3,0.4,0.4) >$, B $=< x,(0.3,0.4,0.4),(0.2,0.2,0.4) >$, C $=< x,(0.4,0.4,0.3),(0.4,0.5,0.4) >$. Then $\varphi = \{\tilde{0},\tilde{1},A,B\}$ is IF topology on X and C is an IFsomewhere dense set.

Theorem: 2.1

If σ is an IFsomewhere dense set in an IFTS (Y,T) ∃ an IF regular closed set γ in (Y,T) such that γ ≤ cl(σ).

Proof:

Let σ be an IF somewhere dense set in (Y,T) and ∃ a non – zero IF open set δ in (Y,T) such that $\delta \leq$ cl(σ). then cl(δ)≤ cl(cl(σ)). Since σ is an IF open set in (Y,T), the closure of δ is an IF regular closed set in (Y,T). Let cl(δ) = γ. Therefore, for IF somewhere dense set σ in (Y,T), such that γ ≤ cl(σ).

Theorem: 2.2

If σ is an IFsomewhere dense set in an IFTS(Y,T), then

- Int(σ) is an IF dense set in (Y,T).
- ∃ an IF regular closed set γ in (Y,T) such that int σ > γ.

Proof:

- Let σ be an IF somewhere dense set in (Y,T).Then 1-σ is an Let σ be an IFcs dense set in (Y,T).Therefore intcl(1-σ)= 0 in (Y,T). Hence 1-cl(int(σ)) ≠ 0 and cl(int(σ))≠1. Thus int(σ) is an IF dense set in (Y,T).
- From (i), int(σ) is an IF dense set in (Y,T). and hence ∃ a fuzzy closed set δ in (Y,T) such that int(σ) > δ. Then int(int(σ)) > int(δ). Hence int(σ) > int(δ) in (Y,T).Since δ is an IFclosed set in (Y,T) and int(δ) is an IFregular open set in (Y,T). Let γ = int(δ).Hence there exists IF regular open set γ in (Y,T) such that int(σ) > γ.

Proposition: 2.1

If σ is a non – zero IF β - open set in an IFTS (Y,T), then σ is an IF dense set in (Y,T).

Proposition: 2.2

If σ and γ are IFsomewhere dense sets in an IFTS(Y,T), then $\sigma \vee$ γ is an IFsomewhere dense set in (Y,T).

Proposition: 2.3

If σ and γ are IFcs dense sets in an IFsomewhere dense sets in (Y,T).Then (1- ($\sigma \wedge$γ)) is an IF somewhere dense sets in (Y,T).

Proof:

Let σ and γ are IF cs dense sets in (Y,T). Then 1- σ and 1- γ are IFsomewhere dense set in (Y,T).1- σ and 1- γ are IFsomewhere dense sets in (Y,T), int(cl(1- σ) ≠ 0

int(cl(1- σ) ≠ 0, in (Y,T).

Therefore, intcl(1- ($\sigma \wedge$γ)) = intcl[(1-σ)$\vee$ (1- γ)]

= int(cl[(1-σ)$\vee$ (1- γ)])

= int[(cl(1-σ))$\vee$(cl (1- γ))]

≠ 0

Then, (1- ($\sigma \wedge \gamma$)) is an IF somewhere dense sets in (Y,T).

Theorem: 2.3

If σ is an IFsimply* set in an IF (Y,T), then σ is an IFsomewhere dense set in (Y,T).

Proof:

Let σ is anIF simply* set in (Y,T). Then $\sigma = \delta \vee \gamma$, where is a non – zero IFopen set and γ is an IFnowhere dense set in (Y,T).

Intcl(σ) = intcl($\delta \vee \gamma$) = int(cl($\delta \vee \gamma$)) ≥ intcl(δ)∨ intcl(γ). Since γ is an IF nowhere dense set in (Y,T), intcl(γ) = 0. Thus intcl(σ) ≥ intcl(δ) ≥int(δ) =δ. but δ is a non – zero IF open set, hence, intcl(σ) ≠0. Therefore, σ is an IFsomewhere dense set in (Y,T).

Theorem: 2.4

Let S and P be IF topological spaces such that S is product related to Y. If σ is an

intuitionistic fuzzy somewhere dense set in S and γ is an IF somewhere dense set in P, then the product$\sigma \times \gamma$ is an IF some where dense set in S × P.

Proof:

Let σ be an IFsomewhere dense set in S and γ is an IFsomewhere dense set in P. Then cl(int(σ)) ≠ 0 in (S,T_1) and cl(int(γ)) ≠ 0 in (P, T_2). Since X is product related to P, intcl($\sigma \times \gamma$) = int(cl (σ) × cl(γ)) = intcl (σ) × int cl(γ)≠0×0≠0. Therefore, the product $\sigma \times \gamma$ is an IF somewhere dense set in the product Space S ×P.

Theorem: 2.5

If σ be an IFsomewhere dense set in (X,T) is an IF P – space then σ is an IFsomewhere dense set in (X,T).

Proof: obvious

Proposition: 2.5

If σ is an IF somewhere dense set in an IFperfectly disconnected space (Y,T), then cl(σ) is an IF pre – closed set in (Y,T)

Proof:

Let σ be an IF somewhere dense set in (X,T). then, intcl(σ) ≠0 in (Y,T). intcl(σ) ≤ cl(σ) implies intcl(σ) ≤ 1 – [1-cl(σ)] in (Y,T). and since an IF topological space (Y,T) is disconnected, cl[intcl(σ)] ≤ 1 –cl [1-cl(σ)] then cl[intcl(σ)] ≤ 1 – [1-intcl(σ)]

This implies that cl[intcl(σ)] ≤intcl(σ), but intcl(σ) ≤ cl[intcl(σ)]. Then cl[intcl(σ)] = intcl(σ) in (Y,T). Therefore, clint[cl(σ)] ≤ cl(σ), implies that cl(σ) is an IF pre- closed set in (Y,T).

Preposition: 2.6

If σ is an IFcs dense set in an IFTS(Y,T), then clint(σ) ≠ 1 in (Y,T).

Theorem: 2.6

An IFset σ defined on X in an IFTS(Y,T) is an IF cs dense set if and if only if there exists an IF closed set γ in (Y,T) suchthat int (σ) ≤γ.

Proof:

Let σ be an IF cs dense set in (Y,T). clint (σ) ≠1 in (Y,T). hence int(σ) is not an IF dense set in(Y,T) and there exists an IFclosed set γ in (Y,T) such that int(σ) ≤ γ <1.

Conversely, suppose that δ is an IFset defined on X such that int(δ) ≤ μ, where 1-δ ∈ T and μ≠ 1. then 1-int(δ)≥1- μ. This implies that cl(1-δ)≥ 1- μ and intcl(1- δ) ≥ int(1- μ) = 1- μ ≠0. Hence 1- μ is an IFsomewhere dense set in (Y,T) and δ is an IFcs dense set in (Y, T).

3. INTUITIONISTIC FUZZY SOMEWHERE CONTINUOUS FUNCTIONS

A function g: (Y,S) →(Z,P) from an IFTS (Y,S) from an IFTS (Z, P), is called an IF somewhere continuous function if whenever int(σ) ≠0 for an IF set σ defined on Z, then $g^{-1}(\sigma)$ is an IF somewhere dense set in (Y,T). That is, g: (Y,S) →(Z,P) ia an IF somewhere continuous function if intcl[$g^{-1}(\sigma)$]≠0 in (Y,T) whenever int(σ) ≠0 for an IF setσ defined on Z.

Example: 3.1

Let X={a,b,c}, Y= {1, 2, 3}and A and B are IFsubsets in X, where

A $= < x, (0.3, 0.1, 0.4), (0.3, 0.4, 0.4) >$, B $= < x, (0.3, 0.4, 0.5), (0.2, 0.2, 0.4) >$, C $= < y, (0.4, 0.4, 0.3), (0.4, 0.5, 0.4) >$. Then $\varphi = \{\tilde{0}, \tilde{1}, A, B\}$ of IFset's in X is an IF topology on X and $\mu = \{\tilde{0}, \tilde{1}, C\}$ of IFset's in Y is an IF topology on Y. Define the function g: X → Y by g(a) = 3, g(b) = 1, g(c) = 2 then $g^{-1}(C) = < x, (0.3, 0.4, 0.4), (0.4, 0.4, 0.5) >$, then $g^{-1}(C)$ is an IFsomewhere continuous function.

Theorem: 3.1

If g: (Y,S) → (Z,P) is an IF continuous function from a IFTS(Y,S) into another IFTS(Z,P), then g is an IFsomewhere continuous function.

Proof:

Let σ be a nonzero set IF defined on Y with int(σ)≠0 in (Y,S). Since the function g is an IF continuous function, g^{-1}(int(σ)) is an IFcontinuous function, then Int(g^{-1}(int(σ)) = g^{-1}(int(σ)) ≠ 0, But g^{-1}(int(σ)) ≤ $g^{-1}(\sigma)$ in (Y,S) and then g^{-1}(int(σ)) ≤ $g^{-1}(\sigma)$ ≤cl(g^{-1}(int(σ)) ≤ cl($g^{-1}(\sigma)$) implies that int(g^{-1}(int(σ))) ≤ int(cl($g^{-1}(\sigma)$)) and int(g^{-1}(int(σ))) ≠0 in (Y,S). Hence g is an IFsomewhere continuous function.

Preposition: 3.1

If g: (Y,S) → (Z,P) is an IF somewhere continuous function from an IFTS(Y,S) into IFTS(Z,P) and if σ is an intuitionistic fuzzy set defined on Z with int(σ) ≠0 in (Z,P), then there exists an IFregular closed set γ in (Y,S) such that γ ≤[cl($g^{-1}(\sigma)$)].

Theorem: 3.2

If g: (Y, S) $\to$(Z, P) is an IFsimply* continuous function from an IFTS(Y, S) into another IFTS (Z, P), then g is an IFsomewhere continuous function from (Y, S) into (Z, P).

Proof:

Let σ be an IFset defined on Z with int(σ)$\neq$0 in (Z, P). Now int(σ) is a nonzero IFopen set in (Z, P). Since g is an IFsimply* continuous function and g^{-1}(int(σ)) is an IFsimply* open set in (Y,S). Then g^{-1}(int(σ)) is an IFsomewhere dense set in (Y, S) and thus intcl[g^{-1}(int(σ))] $\neq$0 in (Y,S) then intcl[g^{-1}(int(σ))] $\leq$ intcl[g^{-1}(σ)] implies intcl[g^{-1}(σ)] $\neq$0 in (Y,S), hence g is an IFsomewhere continuous function.

Theorem: 3.3

If g: (Y, S) $\to$(Z, P) is a somewhere IFcontinuous function and one-to- one function from an IFTS (Y, S) onto IFTS(Z, P) and if σ is an IF open and IF dense set in (Y, S) then g(σ) is an IFdense set in (Z, P).

Proof:

Let σ be an IF open and IF dense set in (Y, S). It is to be proved that g(σ) is an IF dense set in (Z, P). Assume the contrary, suppose that g(σ) is not an IF dense set in (Z, P). That is cl[g^{-1}(σ)] $\neq$1 in (Z, P) and then 1- cl[g^{-1}(σ)] $\neq$0 implies int(1- g(σ)) $\neq$0. Since f is one – one and onto. g(1- σ) = 1- g(σ) and then int[g(1- σ)] $\neq$0 in (Z, P). Since the function g is an IF somewhere continuous function (Y, S) onto (Z, P), g^{-1}(g(1- σ)) is an IFsomewhere dense set in (Y,S). Then intcl(g^{-1}(g(1- σ))$\neq$0. Since g is one to one, g^{-1}(g(1- σ)) = 1- σ and intcl(g^{-1}(g(1- σ))) = intcl(1- σ) $\neq$0. Then 1- clint(σ) $\neq$ 0 in (Y, S) implies clint(σ) $\neq$1and then cl(σ) $\neq$1, a contradiction to σ being an intuitionistic fuzzy dense set in (Y,S). Hence, g(σ) is an IF dense set in (Z, P).

Theorem: 3.4

If g: (Y, S) $\to$(Z, P) is an IF somewhere continuous function and one- to- one function from an IF hyperconnected space (Y, S) onto IFTS (Z, P) and if σ is an IF open in (Y, S) then g(σ) is an IF dense set in (Z, P).

Proof:

If g: (Y, S) $\to$(Z, P) is an IFsomewhere continuous function and one-to- one function from an IF hyperconnected space (Y, S) onto IFTS (Z, P) and if σ is an IF open in (Y, S). Since (Y, S) is an IF hyperconnected space, An IFopen and IFdense set (Y, S) then g(σ) is an IF dense set in (Z, P).

Theorem: 3.5

If g: (Y, S) $\to$(Z, P) is an IFsomewhere continuous function and one- to- one function from an IFhyperconnected space (Y, S) onto IFsubmaximal space (Z, P), then g is an IF open function (Y, T) onto (Z, S).

Proof:

Let σ be an IFopen set in (Y, S). Since (Y, S) is an IFhyperconnected space, An IFopen set σ is an IFdense set in (Y, S). Since, g: (Y, S) $\to$(Z, P) is an IFsomewhere continuous function and one- to- one function from (Y, T) onto (Z, S). Since (Z, S) is an IF submaximal space, IFdense set g(σ) is an IFopen set in (Z, P) and hence g is an IF open function (Y, T) onto (Z, S).

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